



Semester Two Examination, 2016

Question/Answer Booklet

**MATHEMATICS
SPECIALIST
UNITS 3 AND 4**

**Section One:
Calculator-free**

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

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In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	12	12	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula Sheet is not to be handed in with your Question/Booklet.

Section One: Calculator-free

35% (52 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1

(5 marks)

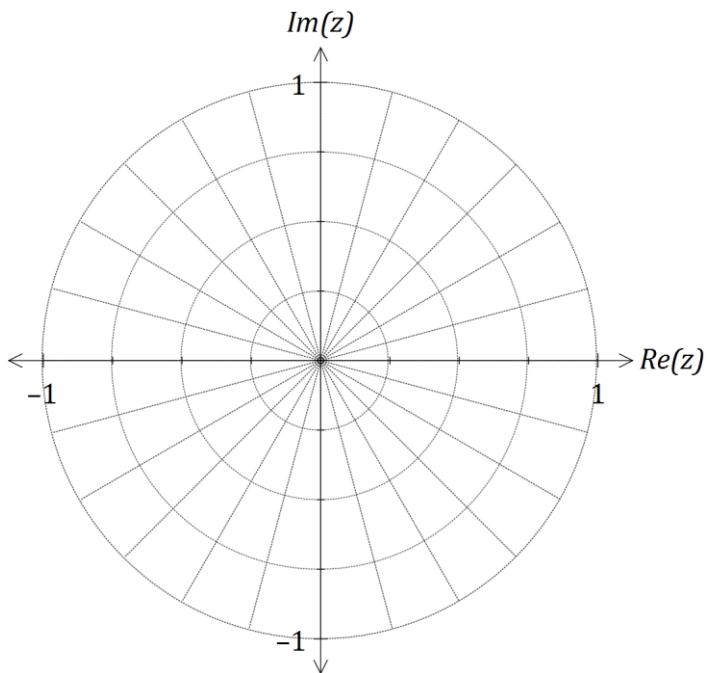
Let $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

(a) Express v in polar form.

(3 marks)

(b) Plot the roots of $z^4 + 1 = 0$ on the following Argand diagram.

(2 marks)



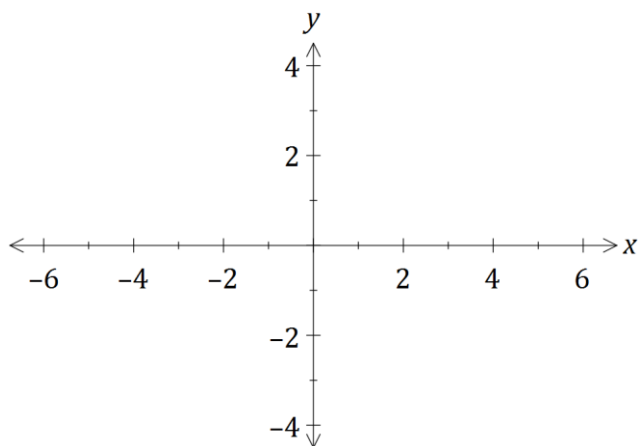
Question 2

(8 marks)

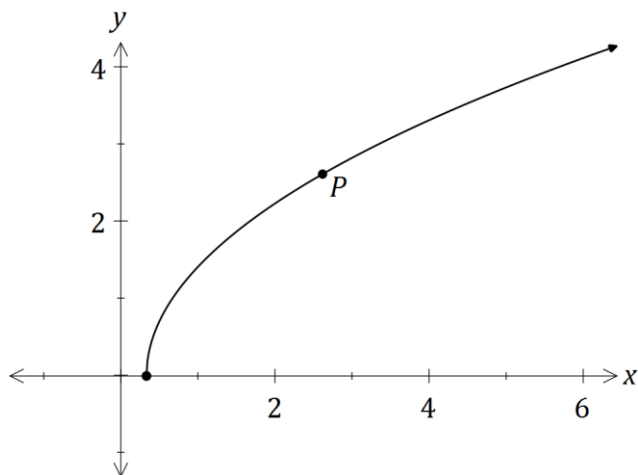
Two functions are defined by $f(x) = \sqrt{3x-1}$ and $g(x) = \frac{1}{x}$.

- (a) Determine the composite function $f(g(x))$ and the domain over which it is defined. (3 marks)

- (b) Sketch the graph of $y = g(g(x))$ on the axes below. (2 marks)



- (c) The graph of $y = f(x)$ is shown below, passing through point P with coordinates $(2.62, 2.62)$. Determine $f^{-1}(x)$, the inverse of $f(x)$, and sketch the graph of $y = f^{-1}(x)$ on the same axes. (3 marks)



Question 3**(5 marks)**

An object, initially at rest, is dropped from the top of tall building so that after t seconds it has velocity v meters per second.

The air resistance encountered by the object is proportional to its velocity, so that the velocity satisfies the equation $\frac{dv}{dt} = 10 - kv$, where k is a constant.

- (a) Express the velocity of the object in terms of t and k . (4 marks)

- (b) Sensors on the object indicate that its velocity will never exceed 55 metres per second. Determine the value of the constant k . (1 mark)

Question 4**(5 marks)**

The polynomial $h(z) = z^4 - 6z^3 + 3az^2 - 30z + 10a$, where a is a real constant, has a zero of $3 - i$. Determine the value of a and all other zeros of $h(z)$.

Question 5

(8 marks)

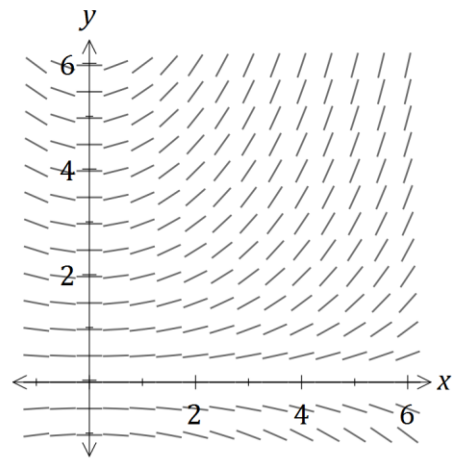
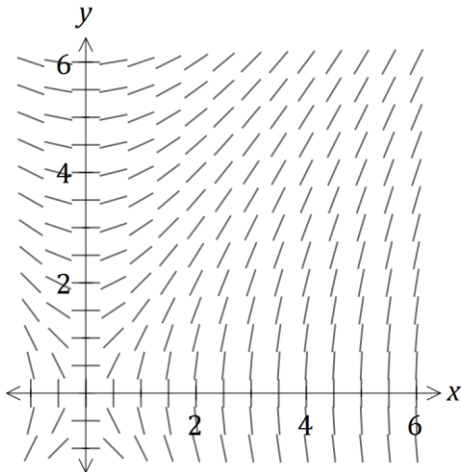
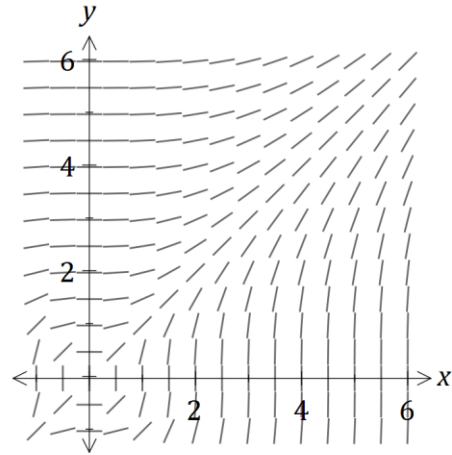
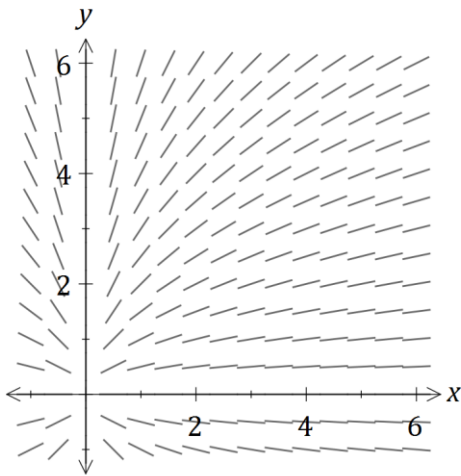
- (a) Using partial fractions, or otherwise, determine $\int \frac{x-19}{(x+1)(x-4)} dx$. (4 marks)

- (b) Use the substitution $u = \sin x$ to evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx$. (4 marks)

Question 6

(5 marks)

The differential equation $y' = \frac{2x}{y}$ is shown in just one of the four slope fields below.



- (a) On the slope field for $y' = \frac{2x}{y}$, sketch the solution of the differential equation that passes through the point $(2, 4)$. (3 marks)
- (b) Another solution to the differential equation passes through the point $(6, -3)$. Use the incremental formula $\delta y \approx \frac{dy}{dx} \times \delta x$, with $\delta x = \frac{1}{10}$, to estimate the y -coordinate of this curve when $x = 6.1$. (2 marks)

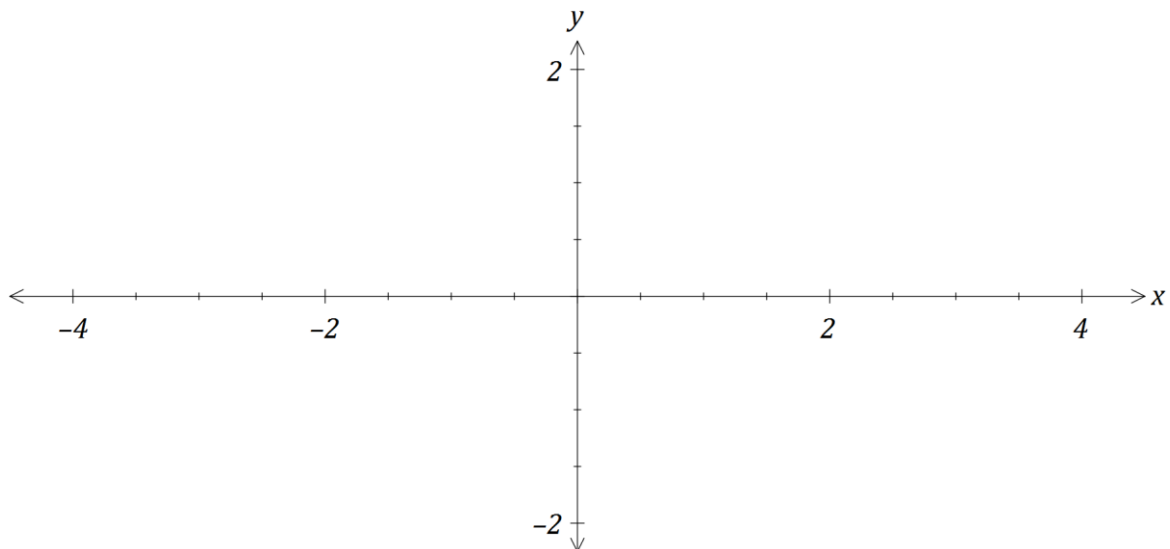
Question 7

(9 marks)

The function f is defined as $f(x) = \frac{x^2-1}{x^2+1}$.

- (a) Show that the **only** stationary point of the function occurs when $x = 0$. (3 marks)

- (b) By considering your work from part (a), the intercepts and the behaviour of the function as $x \rightarrow \pm\infty$, sketch the graph of $y = f(x)$ on the axes below. (3 marks)



- (c) Using your graph, or otherwise, determine all solutions to

(i) $f(x) = |f(x)|$. (1 mark)

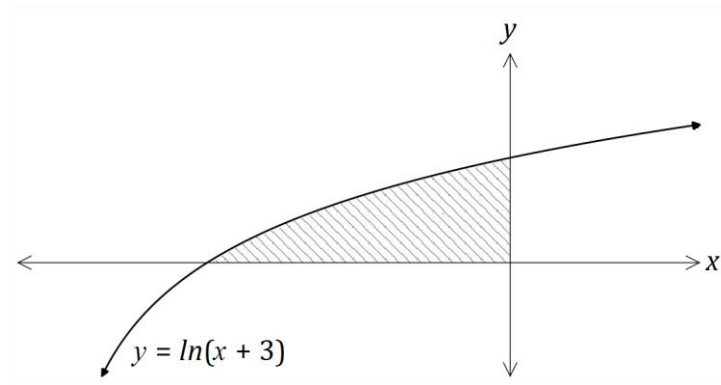
(ii) $f(x) = f(|x|)$. (1 mark)

(iii) $f(x) = \frac{1}{f(x)}$. (1 mark)

Question 8

(8 marks)

A region is bounded by $x = 0$, $y = 0$ and $y = \ln(x + 3)$ as shown in the graph below.



- (a) Show that the area of the region is given by $\int_0^{\ln 3} (3 - e^y) dy$. (3 marks)
(Do not evaluate the integral).

- (b) Determine the volume of the solid generated when the region is rotated through 2π about the y -axis. (5 marks)

Additional working space

Question number: _____

